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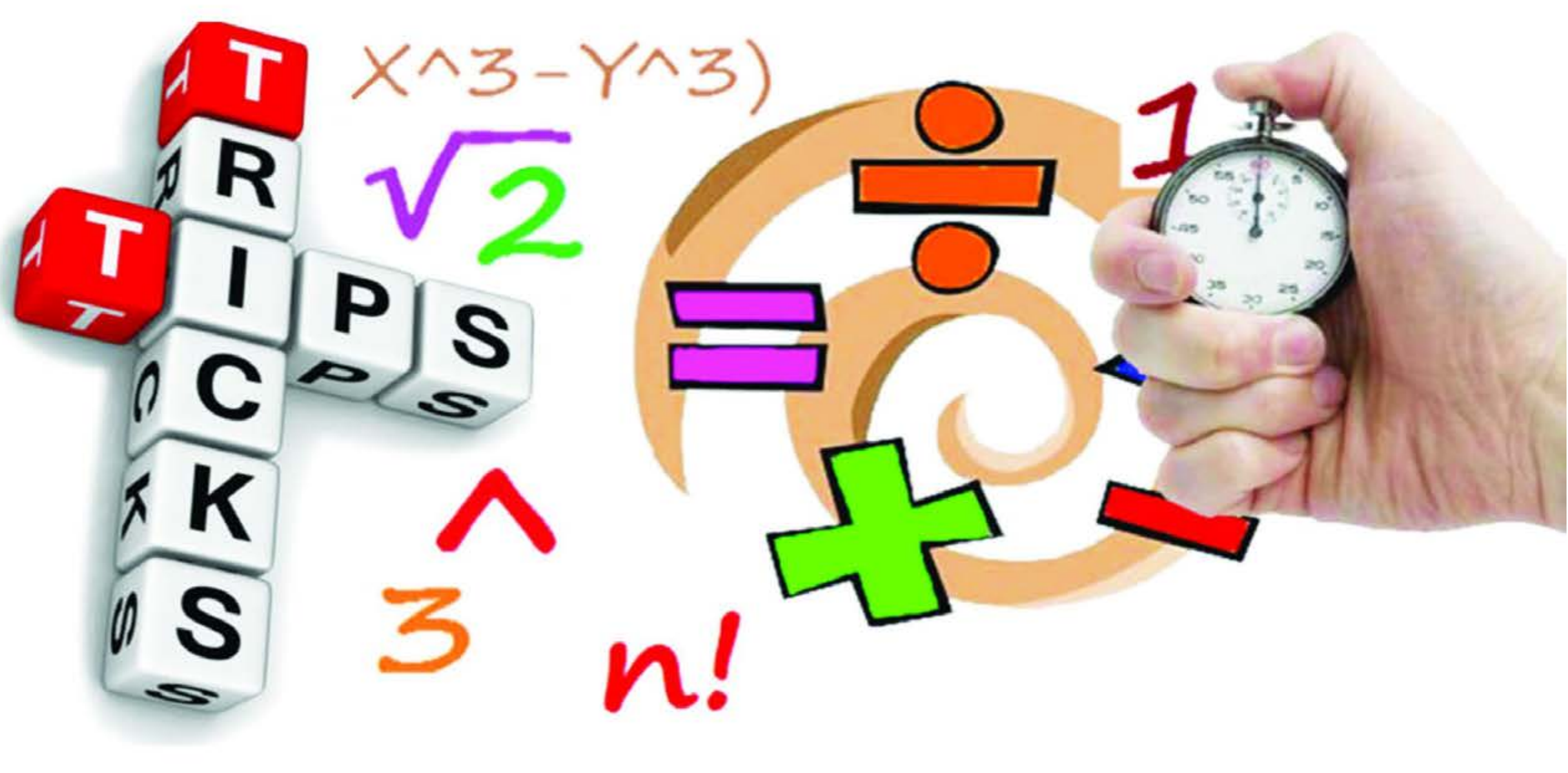
Distance Learning Programme

UPSC Prelims

CSAT
Basic Numeracy



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CSAT BASIC NUMERACY

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CHAPTER 1

NUMBER SYSTEM & SIMPLIFICATION

Number System

The number system which we use at present is called Decimal System. 10 symbols are used in this system *i.e.*, 0, 1, 2, 3, 4, 5, 6, 7, 8, 9.

In Decimal System

1. When we move leftward in reading a number we name it unit, ten, hundred, thousand, ten thousand, lac etc.

5	8	8	8	8	8
↓	↓	↓	↓	↓	↓
Lac	Ten Thousand	Thousand	Hundred	Ten	Unit

2. Therefore the value of left digit is ten time more than the value of its right.

8	8	8	8	= Eight thousand Eight Hundred Eighty Eight
↓	↓	↓	↓	
Eight Thousand	Eight Hundred	Eighty	Eight	

Hence, any number has two values.

(A) Real or imprint value. This is real value of number which can be any number between 0 to 9. It never changes.

(B) Positional Value. The value of a number due to its special position in a digit called its positional value. **For example.** In 53834 the real value of both 3 are 3 but positional value of 3 at tenth position is 30 and at thousand is 3000. Hence positional value of any number can be obtained this way:

8	8	8	8	8	8	8	8	8	8
↓	↓	↓	↓	↓	↓	↓	↓	↓	↓
Arab	10 Crore	Crore	10 Lacs	Lacs	Ten Thousand	Thousand	Hundred	Ten	Unit
8000000000	800000000	80000000	8000000	800000	8×10000	8×1000	8×100	8×10	8×1
8×10^9	8×10^8	8×10^7	8×10^6	8×10^5	8×10^4	8×10^3	8×10^2	8×10^1	8×10^0

Types of Number

1. **Natural Numbers:** The number that we use to count objects are called Natural Numbers or Cardinal Numbers. For example: 1, 2, 3, 4, 5, 6... etc.

   Note. Zero (0) is not a natural number because we start counting from 1. The smallest or first natural number is 1.
2. **Whole Numbers:** The natural numbers including zero (0) is called whole numbers. For example: 0, 1, 2, 3, 4... etc.
3. **Even Numbers:** Natural numbers divisible by 2 are called even numbers. For example: 2, 4, 6, 8, 10... etc.
4. **Odd Numbers:** Natural numbers not perfectly divisible by 2 are called odd numbers. For example: 1, 3, 5, 7, 9... etc.

$$(\text{Even Number})^n = \text{Even Number}$$

$$(\text{Odd Number})^n = \text{Odd Number}$$

Where n = Natural Number

Natural Number

(Even Number) (Odd Number)

5. **Integers:** Integers include natural numbers, zero and negative numbers. For example: –5, –4, –3, –2, –1, 0, 1, 2, 3, 4... etc.
   Note: Zero is neither negative nor positive number.
6. **Prime Numbers:** Natural numbers more than 1 which are divisible by either 1 or itself only are called prime numbers. For example: 2, 3, 5, 7, 11...
   Or numbers that have only two divisors. For example: $2 = 1 \times 2, 2 \times 1$.
7. **Factor Numbers:** Natural numbers which are divisible by at least 3 natural numbers are called factor numbers. For example: 4, 6, 8, 9, 10, 12, 14, 16... etc.
   $4 = 1 \times 4, 2 \times 2, 4 \times 1, 6 = 1 \times 6, 2 \times 3, 3 \times 2, 6 \times 1$ etc.
8. **Rational Numbers:** Numbers in the form of $\frac{p}{q}$ where p and q are whole numbers and $q \neq 0$ are called national numbers. For example: $\frac{2}{3}, \frac{5}{6}, \frac{3}{5}$ ... etc.
   Note: All Natural numbers are also Rational Numbers as all Natural numbers can be written in the form of $\frac{1}{1}, \frac{2}{1}, \frac{3}{1}$ ... etc.
9. **Irrational Numbers:** Numbers which can't be written in the form of $\frac{p}{q}$ are called irrational numbers. For example : $\sqrt{2}, \sqrt{3}, \sqrt{7}, \frac{\sqrt{11}}{\sqrt{13}}$ ... etc.
   Note: Rational numbers and Irrational numbers are collectively called Real numbers. For example $\sqrt{2}, \frac{3}{8}, \frac{\sqrt{5}}{\sqrt{7}}, 2$ etc.

Real Number

Rational Number Irrational Number

10. **Co-prime number:** Pair of numbers a and b for which H.C.F is 1 are called co-prime numbers. For example: (4, 7), (7, 11), (3, 13) etc.

Decimal Expansion

Finite decimal or Terminating decimal example : 0.5, 0.265 etc.

These numbers can be changed into Rational number by removing decimal, which means these are Rational numbers.

(Repeating decimal or Recurring decimal)

Repeating

Example:

(i) $0.333.... = 0.\overline{3}$

(ii) $0.2656565 ..... = 0.2\overline{65}$

These numbers can also be changed into Rational numbers.

Non-Repeating

Example:

(i) 3.14....

(ii) 1.7325....

These are Irrational numbers.

Let's discuss some important formula after introduction of different types of numbers:

1. Sum of first n Natural Numbers $= 1 + 2 + 3 + 4 + ... + n = \frac{n(n+1)}{2}$
2. Sum of sqaure of first n Natural Numbers $= 1^2 + 2^2 + 3^2 + ... + n^2 = \frac{n(n+1)(2n+1)}{6}$
3. Sum of cube of first n Natural Numbers $= 1^3 + 2^3 + 3^3 + ... + n^3 = \left[\frac{n(n+1)}{2}\right]^2$
4. Sum of first n Even Numbers $= 2 + 4 + 6 + 8 + ... + n = n(n + 1)$
5. Sum of first n odd Numbers $= n^2$

Dividend and Remainder

When a number a is divided by number b, a is called Dividend and b is called Divider. The number of times dividend is divided by the divider is called Quotient and the amount remains at the end is called Remainder. Remainder will be zero (0) if Dividend is absolutely divided by the Divider.

Divider) Dividend (Quotient

Remainder

Divider × Quotient + Remainder = Dividend

Example:

Divider, Dividend, Quotient

4) 54 (13
4
14
12
2 ← Remainder

4 × 13 + 2 = 54

Divisibility Tests

A given number is absolutely divided by some special numbers or not can be tested on following facts.

- A number will be absolutely divided by 2 if its unit is either even number or zero.
- A number will be absolutely divided by 3 if the sum of digits of the number is divisible by 3.
- A number will be absolutely divided by 4 if the last two digits of the number is absolutely divisible by 4.
- A number will be absolutely divided by 5 if its unit is either five or zero.
- A number will be absolutely divided by 6 if it is absolutely divisible by both 2 and 3.
- A number will be absolutely divided by 8 if its last three digits are absolutely divisible by 8.
- A number will be absolutely divided by 9 if sum of its digits is divisible by 9.
- A number will be absolutely divided by 10 if its unit is 0.
- A number will be absolutely divided by 11 if the difference between sum of its even digits and odd digits is either 0 or divisible of 11.

Example 1:

+
1 3 3 1
+

$\Rightarrow$ Difference = 4 – 4 = 0

1331 will be absolutely divided by 11.

Example 2:

$$\overset{+}{\overbrace{5\quad 5}}\quad 3\quad 2 \qquad 5\quad \underset{+}{\underbrace{5\quad 3}}\quad 2$$

$\Rightarrow$ 8 – 7 = 1; since 1 is neither 0 nor divisible of 11, 5532 will not be divided by 11.

- Two numbers a and b are co-prime numbers. If a given number is absolutely divisible by a and b, it will also be absolutely divisible by a × b. Example: 180 is absolutely divisible by 2 and 3, it will also be divisible by 2 × 3 = 6.
- If x^n is divided by (x + 1) then,

  (a) If n = Even Number then Remainder = 1

  (b) If n = Odd Number then Remainder = x

  Example: If 17^{300} divided by 18 then remainder = 1

  If 17^{201} divided by 18 the remainder = 17
- If $(x + 1)^n$ is divided by x then remainder will be always = 1

  Example: If 18^{19} is divided by 17 then remainder = 1
- $(a^n + b^n)$ will be absolutely divided by (a + b) if n is an odd number.
- $(a^n - b^n)$ will be absolutely divided by (a + b) if n is an even number.
- $(a^n - b^n)$, will be always divided by (a – b).

  Note: $(a^n + b^n + c^n + ...)$ will be absolutely divided by (a + b + c ...) if n is an odd number and a, b, c,... are in a parallel series.

Remainder Theorem

(i) Remainder of $\left(\frac{a+b}{n}\right) = \left(\text{Remainder of } \frac{a}{n}\right) + \left(\text{Remainder of } \frac{b}{n}\right)$

(ii) Remainder of $\left(\frac{a \times b}{n}\right) = \left(\text{Remainder of } \frac{a}{n}\right) \times \left(\text{Remainder of } \frac{b}{n}\right)$

Note: If remainder is more than n, it will be further divided by n.

Example: Find Remainder of $\frac{12 \times 13 \times 15 \times 16}{7}$.

According to Remainder Theorem

Remainder = Remainder of $\left(\frac{12}{7}\right)$ × Remainder of $\left(\frac{13}{7}\right)$ × Remainder of $\left(\frac{15}{7}\right)$ × Remainder of $\left(\frac{16}{7}\right)$

= 5 × 6 × 1 × 2 = 60

Since 60 is more than 7, the actual remainder will be $\left(\frac{60}{7}\right)$ = 4.

Finding the Unit Number

First Method: The digit of unit number depends on the last digit of that number. Let's find it with an example:

Example 1: Find unit number of $(2014)^{2014}$

Answer. First $(2014)^{2014} \Rightarrow (4)^{2014}$

(Since unit number depends on the last digit)

Now the remainder of exponent/4 will be the new exponent on last digit.

$= 4^2 = 1\boxed{6} \Rightarrow$ unit number = 6.

These questions can be solved in the following steps:

(i) First of all number will be replaced by its unit number.

(ii) The exponent will be replaced by its last two digit and the new exponent obtained in this way will be divided further by 4.

(iii) The remainder found in this way will be put as exponent and the unit number thus found will be the unit number of whole question.

Note: If remainder found in this way = 0 then exponent will be 4.

Example 2: $(2527)^{256} \Rightarrow (7)^{256}$

$\Rightarrow (7)^{56} \Rightarrow (7)^{56/4} \Rightarrow (7)^4$

$\Rightarrow 2401 \Rightarrow$ unit number will be 1

Second Method.

1. Any exponent on 1, the unit number will be 1.
2. The unit number of 3^2 will be 9, 3^3 will be 7 and 3^4 will be 1.
3. If exponent on 4 is an even number the unit number will be 6 and if exponent on 4 is an odd number the unit number will be 4.
4. Any exponent on 5, the unit number will be 5.
5. Any exponent on 6, the unit number will be 6.
6. Unit number of 7^4 will be 1.
7. If exponent on a negative number is an even number then answer will be a positive number and if exponent is an odd number then answer will be a negative number.

In short,

(1)	$1^{\text{any exponent}} = 1$	(2)	$3^4 = 1 = 81$
(3)	$4^{\text{even number}} = 6$	(4)	$4^{\text{odd number}} = 4$
(5)	$5^{\text{any exponent}} = 5$	(6)	$6^{\text{any exponent}} = 6$
(7)	$7^4 = 1 = 2401$	(8)	$(\text{–ve number})^{\text{even number}} = +\text{ve value}$
(9)	$(\text{–ve number})^{\text{odd number}} = \text{–ve value}$		

Example: Find the unit digit of $(2497)^{248} \times (323)^{726}$

$= (7)^{248} \times (3)^{726}$

$= (7^4)^{62} \times (3^4)^{181} \times 3^2$ $[(a^m)^n = a^{m\times n}$ & $a^m \times a^n = a^{m+n}]$

$= (1)^{62} \times (1)^{181} \times 9 = 1 \times 1 \times 9 = 9$

Finding the Number of Factors

Number of factors means the total number of those number which can absolutely divide the given number. For example: If we talk about the number of divisors of 6, it is 1, 2, 3 and 6. Hence total number of divisors = 4. But it will be difficult to find out the number of divisors of a large number. It will be obtained by the following formula.

If $N = 2^x \times 3^y \times 5^z \times ...$

The number of divisors of $N = (x + 1)(y + 1)(z + 1)$

Example 1: If N = 540

$\Rightarrow$ $N = 2^2 \times 3^3 \times 5^1$

$\Rightarrow$ No. of divisors = (2 + 1) (3 + 1) (1 + 1)

= 3 × 4 × 2 = 24

2	540
2	270
3	135
3	45
3	15
5	5
	1

Example 2: If N = 1650

$\Rightarrow$ $N = 2^1 \times 3^1 \times 5^2 \times 11^1$

$\Rightarrow$ No. of divisors = (1 + 1) (1 + 1) (2 + 1) (1 + 1)

= 2 × 2 × 3 × 2 = 24

Fraction

Numbers written is the form of $\frac{p}{q}$ where p and q are Whole numbers and $q \neq 0$ are called fraction numbers. In other words, final value obtained by dividing a number by another is called fraction.

In $\frac{p}{q}$, p is numerator and q is denominator.

Types of Fraction

1. **Proper Fraction:** Fraction whose numerator is less than the denominator is called proper fraction. For example: $\frac{1}{5}, \frac{11}{19}, \frac{20}{35}$ etc.
2. **Improper Fraction:** Fraction with the numerator either equal to or greater than the denominator is called improper fraction. For example: Every Natural number, $\frac{5}{3}, \frac{19}{11}, \frac{35}{20}$ etc.
3. **Mixed Fraction:** A combination of a proper fraction and a Whole number is called a mixed fraction. For example: $1\frac{1}{3}, 2\frac{3}{4}, 5\frac{7}{8}$ etc.
4. **Decimal Fraction:** Fractions whose denominators are 10, 100 or 1000 etc. are called decimal fraction. For example: $\frac{1}{10}, \frac{9}{100}, \frac{11}{10}$ etc.

Comparison of Fractions (Increasing and Decreasing order)

1. If denominators of fractions are equal then the fractions with greater numerator will be the greater. For example: $\frac{7}{5} > \frac{3}{5} > \frac{1}{5}$
2. If numerators of fractions are equal then the fraction with smaller denominator will be the greater. For example: $\frac{7}{5} > \frac{7}{9} > \frac{7}{11}$
3. If the differences of numerator and denominator are equal in the given factors then
   (a) Factor having smaller numerator will be greater if numerator is greater than denominator.
   For example: $\frac{7}{5} > \frac{9}{7} > \frac{11}{9} > \frac{13}{11}$
   (b) Factor having greater numerator will be greater if denominator is greater than numerator.
   For example: $\frac{11}{13} > \frac{9}{11} > \frac{7}{9} > \frac{5}{7}$

Simplification

Questions related to simplification were not asked directly but basic rules and useful formulas can be helpful to solve related questions in short time.

BODMAS

The steps to solve an arithmetic equation.

B = Bracket:

Linear Bracket	Small Bracket	Middle Bracket	Large Bracket
ex. $\overline{2+3}$	ex. (2+3)	ex. {2+3}	ex. [2+3]

O = Of, it generally functions as multipication but in an equation it is executed before division.

D = Division

M = Multiplication

A = Addition

S = Subtraction

Indices

If a number a is multiplied in n term by itself it can also be written as a^n. In a^n, a is called base and n is called index. For example: $2 \times 2 \times 2 \times 2 \times 2 = 2^5$

Note:

1. $a^m \times a^n = a^{m+n}$
2. $\frac{a^m}{a^n} = a^{m-n}$
3. If $a^x = a^y$ then $x = y$
4. If $a^x = b^x$ then $a = b$
5. $(a^m)^n = a^{m \times n} = (a^n)^m$
6. $a^{-m} = \frac{1}{a^m}$
7. $\left(\frac{a}{b}\right)^{-m} = \left(\frac{b}{a}\right)^m$
8. $a^0 = 1$, $a^1 = a$

EXERCISE

1. What will be the sum of all Natural numbers from 46 to 92?
   (a) 3243 (b) 2743
   (c) 4536 (d) 1833
2. The sum of square of two numbers is 146. If the square of their difference is 36 then the product of both number is:
   (a) 72 (b) 160
   (c) 55 (d) 16
3. The difference of a two digit number and the number formed by changing the digit is 54. If the sum of its digit is 8 then the original number will be:
   (a) 62 (b) 53
   (c) 71 (d) 80
4. The sum of two numbers is 17 and product is 72. What will be the sum of its reciprocals.
   (a) $\frac{17}{72}$ (b) $\frac{1}{7}$
   (c) $\frac{1}{17}$ (d) $\frac{17}{89}$
5. What will be the difference between the sum of first 25 even numbers and the sum of first 25 odd numbers?
   (a) 50 (b) 25
   (c) 125 (d) 250
6. When a number is divided by 44 the remainder is 27. What will be the remainder if this number is divided by 11?